MATHEMATICS METHODS : UNITS 1 & 2, 2020

Test 4 –Sequences, Series, Differentiation (10%)

(2.2.1 to 2.2.9, 2.3.1 to 2.3.22)

Calculator Free - Allow 1 Minute of Reading Time

Time Allowed 20 Minutes	First Name	Surname	Marks 20 marks
	Solutions		

Circle your Teacher's Name: Bestall Goh Fraser-Jones Freer Koulianos Luzuk Rudland Tanday

Assessment Conditions: ***(N.B. Sufficient working out must be shown to gain full marks)***

- Calculators: Not Allowed
- Formula Sheet: Provided
- Notes: Not Allowed

Question 1 **[3, 2 = 5 marks]**

An arithmetic sequence is such that: $T_{19} = 61$ and $T_{30} = 94$,

(a) Determine the first three terms of the sequence, and

$T_{19} = a + 18d = 61$ ①

$T_{30} = a + 29d = 94$ ②

② − ① $\quad 11d = 33$

$\therefore d = 3$ ✓

$\therefore a = 7$ ✓

$\Rightarrow T_1 = 7$

$T_2 = 10$ ✓

$T_3 = 13$

(b) Determine S_{30} ?

$S_{30} = \frac{n}{2}[2a + (n-1)d]$

$\therefore S_{30} = \frac{30}{2}[2(7) + 29(3)]$ ✓

$\therefore S_{30} = 15[14 + 87]$

$\therefore S_{30} = 1015$ ✓

OR

$S_{30} = \frac{n}{2}(a + l)$

$\therefore S_{30} = \frac{30}{2}(7 + 94)$ ✓

$\therefore S_{30} = 15 \times 101$

$\therefore S_{30} = 1015$ ✓

Question 2 **[3 marks]**

Differentiate using the first pinciple definition $y = 3x^2 - 1$.

$$\text{gradient} = \lim_{h \to 0} \frac{(3(x+h)^2 - 1) - (3x^2 - 1)}{x + h - x} \quad \checkmark$$

$$= \lim_{h \to 0} \frac{3x^2 + 6xh + 3h^2 - 1 - 3x^2 + 1}{h}$$

$$= \lim_{h \to 0} \frac{6xh + 3h^2}{h} \quad \checkmark$$

$$= \lim_{h \to 0} \frac{h(6x + 3h)}{h}$$

$$= 6x \quad \checkmark$$

Question 3 **[1, 2, 3 = 6 marks]**

(a) Determine $\frac{dy}{dx}$ for the following, leaving your answer in simplest form, with positive indices:

(i) $y = 5x^4$

$\therefore \frac{dy}{dx} = 20x^3$ ✓

(ii) $y = 12(\sqrt{x})^3$

$y = 12x^{3/2}$

$\therefore \frac{dy}{dx} = 12 \times \frac{3}{2} x^{\frac{1}{2}}$ ✓

$\therefore \frac{dy}{dx} = 18x^{\frac{1}{2}}$ ✓

(b) Determine $\frac{dy}{dx}$ when $x = 1$, if $y = \frac{x^4}{4} - \frac{2}{x^2}$.

$\therefore y = \frac{x^4}{4} - 2x^{-2}$ ✓

$\therefore \frac{dy}{dx} = x^3 + 4x^{-3}$ ✓

$\therefore \left.\frac{dy}{dx}\right|_{x=1} = 1^3 + 4(1)^{-3} = 5$ ✓

Question 4 **[4, 2 = 6 marks]**

(a) Use calculus techniques to determine the coordinates of both stationary points of $y = (x+2)^2(x-4)$.

$\therefore y = (x^2 + 4x + 4)(x-4)$

$\therefore y = x^3 - 4x^2 + 4x^2 - 16x + 4x - 16$

$\therefore y = x^3 - 12x - 16$ ✓

$\frac{dy}{dx} = 3x^2 - 12 = 0$ ✓

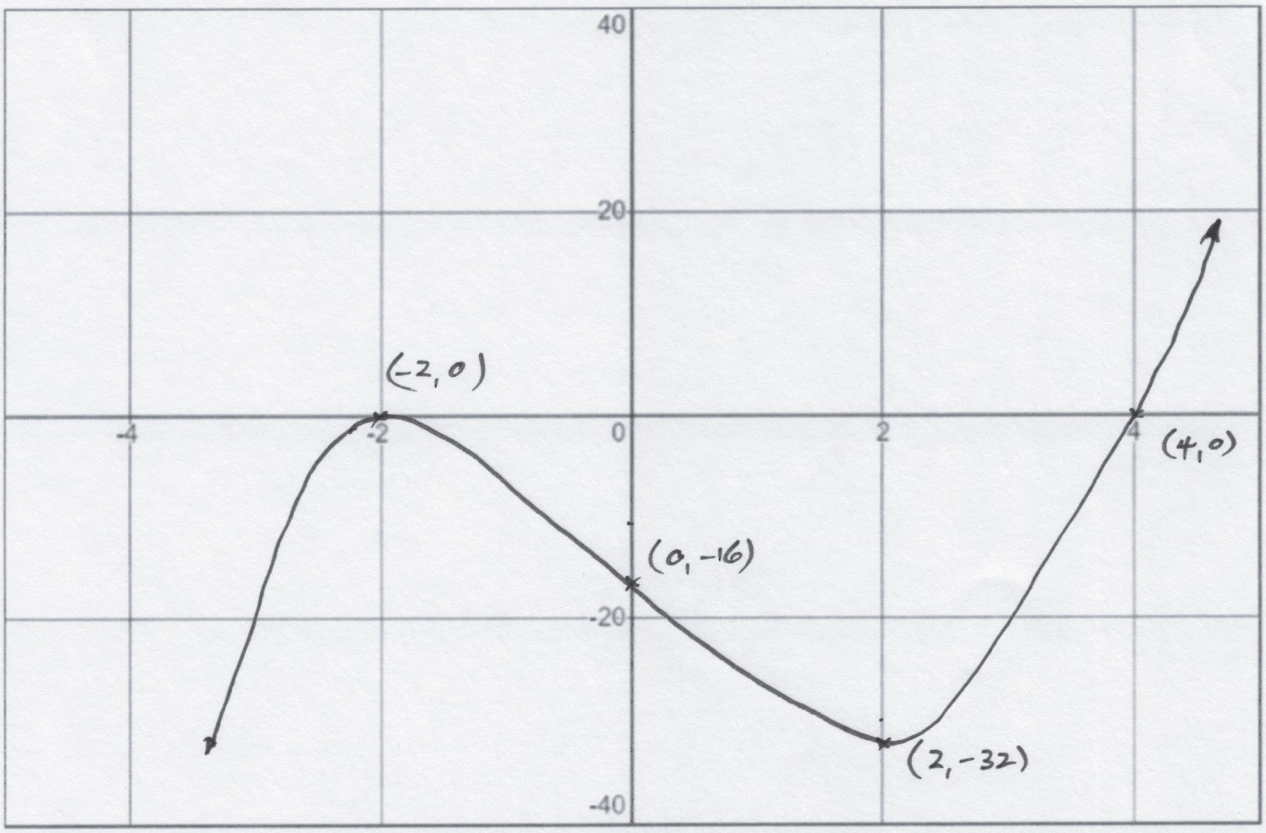
$\therefore 3(x^2 - 4) = 0$

$\therefore 3(x+2)(x-2) = 0$

$\therefore x = -2$ or $x = +2$

$\therefore$ co-ordinates are $(-2, 0)$ ✓ $(2, -32)$ ✓

(b) Graph the function on the axes below showing all axes intercepts and the stationary points:



✓ - general shape
✓ - all points

-END OF CALCULATOR FREE SECTION-

MATHEMATICS METHODS : UNITS 1 & 2, 2020

Test 3 –Sequences, Series, Differentiation (10%)

(2.2.1 to 2.2.9, 2.3.1 to 2.3.22)

Calculator Assumed - Allow 1 Minute of Reading Time

Time Allowed 25 Minutes	First Name	Surname	Marks 27 marks

Circle your Teacher's Name: Bestall Goh Fraser-Jones Freer Koulianos Luzuk Rudland Tanday

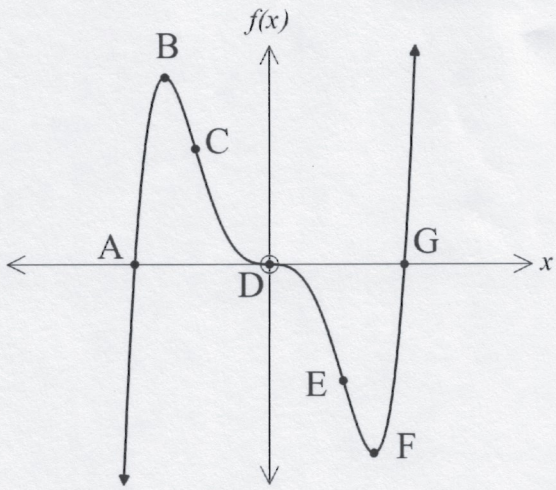
Assessment Conditions: ***(N.B. Sufficient working out must be shown to gain full marks)***

- Calculators: Allowed
- Formula Sheet: Provided
- Notes: Not Allowed

Question 5 **[4 marks]**

Study the diagram on the right.

Hence, name the point(s) using letters A to G,



(i) where $f(x)=0$

A, D, G ✓

(ii) where $f'(x)<0$

C, E ✓

(iii) where $f'(x)=0$ and there is a change in concavity.

D ✓

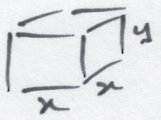
(iv) which would be the x intercepts on the graph of $y=f'(x)$.

B, D, F ✓

Question 6 **[3, 5 = 8 marks]**

A closed rectangular block has a square base of side, x cm, and a height of y cm, and a volume $80\,cm^3$. The base and top are to be covered with paint costing $5\ cents/cm^2$ and the sides with paint costing $4\ cents/cm^2$.

(a) Show that the Cost function, C, is given by $C = 10x^2 + \frac{1280}{x}$ where C is measured in cents.



$V = x^2y = 80$

$\therefore y = \frac{80}{x^2}$ ✓

$C = 5 \times 2x^2 + 16 \times \frac{80}{x}$ ✓

$\therefore C = 10x^2 + \frac{1280}{x}$ ✓

(b) Use calculus techniques to find the values for x and y that minimises the cost of painting the block, and state the minimum cost?

$\frac{dC}{dx} = 20x - \frac{1280}{x^2} = 0$ ✓

$\therefore 20x^3 - 1280 = 0$

$\therefore 20(x^3 - 64) = 0$

Class Pad

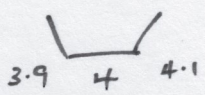
$\therefore x = 4cm$ ✓

$y = 5cm$ ✓

$\therefore$ Minimum Cost $= 10(4)^2 + \frac{1280}{4}$

$= \$4.80$ ✓

SIGN TEST or GRAPH



✓

Question 7 **[3, 4 = 7 marks]**

A farmer estimates that if 75 pecan nut trees are planted per hectare, the average yield per tree will be 7kg.

For every tree less that he plants on the same acreage, the average yield per tree will increase by 0.2 kg per tree.

(a) Complete row 3 in the table below, and hence determine general expressions for **A** and **B** in the table:

Number of Trees	Number of kg	Yield
75	7	525
$75-1$	$7+0.2$	$(75-1)(7+0.2)$
$75-2$	$7+2(0.2)$	$(75-2)(7+2(0.2))$ ✓
$(75-x)$	**A** $7+0.2x$ ✓	**B** $(75-x)(7+0.2x)$ ✓

(b) Use calculus techniques to determine how many trees per hectare should the farmer plant to maximise yield?

$$\text{YIELD} = (75-x)(7+0.2x) = 525+8x-0.2x^2 \quad ✓$$

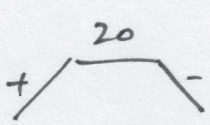
$$\therefore \frac{dy}{dx} = 8-0.4x = 0$$

$$\therefore x = 20 \quad ✓$$

$$\therefore \text{farmer should plant} = 75-20$$

$$= 55 \text{ trees.} ✓$$

SIGN TEST



✓

Question 8 **[2, 2, 2, 2 = 8 marks]**

Thomas the Tank Engine is moving around a circular track of radius 1.4 m and at the end of the first minute had completed 12 laps of the track. In each subsequent minute, as the batteries run down, the train travels 90% of the distance travelled in the previous minute.

(a) Determine the distance travelled by Thomas the Tank Engine during the fifth minute, to two decimal places.

Distance travelled in first minute = $2\pi(1.4)\times 12$

$= 33.6\pi = 105.5575$ m. ✓

$\therefore$ Train travels 69.2563 m. ✓ in the fifth minute

(b) During which minute does Thomas the Tank Engine first travel less than one complete lap of the circuit?

Class Pad → $T_{n+1} = T_n \times 0.90$ $T_1 = 105.5575$ ✓

$\therefore$ 25th minute ✓

(c) Determine the minimum time, to the nearest minute, that Thomas the Tank Engine takes to travel a total distance of 500 metres.

7 minutes ✓✓ (6 mins = 1 mark)

(d) Show that the train will complete more than 115 laps of the circuit.

$S_\infty = \frac{a}{1-r}$

$= \frac{33.6\pi}{1-0.90}$

$= 1055.575$ metres ✓

Distance of 115 laps $= 115 \times 8.7964$ m

$= 1011.586$ m.

$\therefore$ Will complete more ✓ than 115 laps as $S_\infty >$ Total distance of 115 laps.

OR

ALT. (IN LAPS),

$S_\infty = \frac{12}{1-0.9}$

$= 120$ laps ✓

+ statement ✓

-END OF CALCULATOR ASSUMED SECTION-